Thermodynamics I

Quiz 2 12/18/03

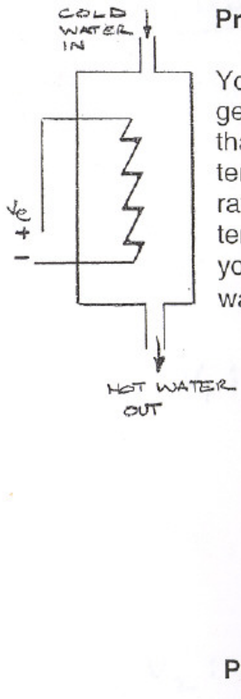
Hello people. Please read the questions carefully. As you solve them, indicate clearly your system/CV boundaries and any assumptions you might make. When possible, please state in words what you are trying to solve/your approach so that I can follow your work and give you credit. This is especially important if you run out of time. Most importantly, relax!

Exam time: 90 minutes
You may use the integral form of the conservation equations throughout

Problem 1

One winter day you are sitting in your kitchen, studying thermodynamics, and shivering cold. You notice a large 5 liter plastic jug of water sitting on the counter, and decide to "heat up" the room by putting it in the refrigerator, which you estimate operates with a COP of 3, and an inside temperature of 4 C. Your brother thinks you're crazy for trying to heat up the kitchen by cooling the water. The kitchen dimensions are 4m x 3m x 4m, and the initial temperature is 20 C.

Esimate the most that the air temperature in the room could change as a result of your putting the jug of water in the refrigerator. Assume air density of 1.2 kg/m3, Cp = 1.0 kJ/kg-K, Cv = 0.7 kJ/kg-K. Assume a water density of 1000 kg/m3, and C = 4.2 kJ/kg-K. 1000 liter = 1 m^3.

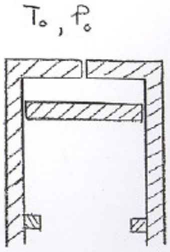


Problem 2

You enjoy taking hot showers. One day as you pondered thermodynamics and how the water gets hot in the first place, you begin analyzing the hot water heater in the bathroom. You notice that it is heated by an electric coil which is controlled by a thermostat setting. When the water temperature drops below the set temperature T_s, the coil turns on and provides electric heat at a rate of W_e watts. As water is drawn out from the bottom of the heater, fresh, cold water at a temperature T_c flows in at the top. After several measurements with a bucket and stop-watch, you determine that the average mass flow rate at which you consume hot water is m kg/s. The water tank has a volume V, the water has a constant specific heat C and density ρ.

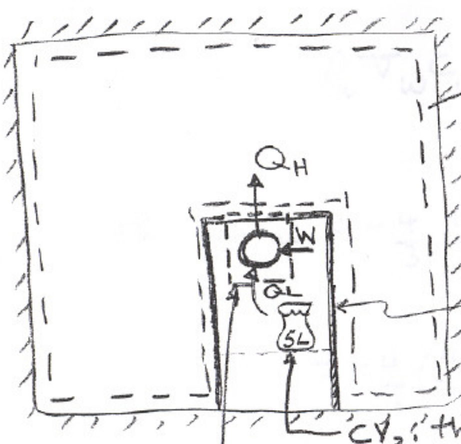
a) If you allow the water to run for a very long time, what will be the steady state temperature of the water exiting the tank, T_e?
b) Having let the tank temperature drop to the steady state temperture by letting the water run for a long time, you close the shower water, and allow the temperature of the water in the tank to begin rising again. How long will it take to get to T_s?
c) Now that the water temperature is back up to T_s, you decide to take a shower with your trusty thermometer. Derive (but do not solve) the differential equation governing how T_e varies in time, in terms of the given variables. Assume that the temperature within the hot water tank is essentially the same everywhere.

Problem 3



A cylinder with its axis vertical and the cylinder head at the top is fitted with a piston. Initially the piston is at the top of the cylinder, with negligible clearance volume between it and the head. The pison is held in position by the atmospheric pressure acting behind it. A small hole in the cylinder head is opened to let air from the atmosphere flow into the cylinder, allowing the piston to drop slowly while the pressure within the cylinder remains constant. The piston and cylinder are made of thermally insulating material. When the piston comes to rest against the stops, the final temperature and volume of air in the cylinder are designated by T_f and V_f. Atmospheric temperature and pressure are T_o and p_o. Determine T_f in terms of the other variables. ALSO, EXPLAIN WHERE THE ENTHALPY OF THE ENTERING AIR WENT DURING THE PROCESS.

PROBLEM I



Q_L is removed from the water to cool it to the refrigerator temperature. The refrigerator requires work W to remove Q_L and rejects Q_H into the air in the room.

assumptions ;
- the refrigerator operates in a cycle
- negligible kinetic & potential energy effects
- the container of the water is thin and has negligible ΔU
- the room is sealed and all the Q_H goes into heating the air since we are interested in the max temperature rise possible
- const densities and specific heats

CV1 : the air

$$Q_H - \cancel{W} + \sum_{in-out} \overset{\text{sealed room}}{\cancel{m}}(e+Pv) = \Delta E = \Delta U$$

$$\underline{Q_H = m_{air} C_{v_{air}} \Delta T_{air} = \rho_{air} V_{air} C_{v_{air}} \Delta T_{air}}$$

CV2 : the refrigerator after we put the water in

$$Q_L - Q_H + W + \sum \overset{\text{sealed fridge}}{\cancel{m}}(e+Pv) = \Delta E = \Delta U = 0$$ since it operates in a cycle.

$$\Rightarrow \quad W = Q_H - Q_L$$

but $COP = \frac{Q_L}{W} = 3 = \frac{Q_L}{Q_H - Q_L} \Rightarrow \underline{Q_H = \frac{4}{3} Q_L}$

CV3 : water

$$-Q_L - \cancel{W} + \sum_{in-out} (e+Pv)\cancel{m} = \Delta E = \Delta U$$

$$-Q_L = \Delta U = \rho_w V_w C_w \Delta T_w$$

$$\Rightarrow \quad \underline{Q_L = -\rho_w V_w C_w \Delta T_w}$$

putting this all together we get:

$$Q_H = \frac{4}{3} Q_L = -\frac{4}{3} \rho_w V_w c_w \Delta T_w = \rho_{air} V_{air} c_{v\,air} \Delta T_{air}$$

$$\Rightarrow \Delta T_{air} = -\frac{4}{3} \frac{\rho_w}{\rho_{air}} \frac{c_w}{c_{air}} \frac{V_w}{V_{air}} \Delta T_w$$

$$= -\frac{4}{3}\left(\frac{1000}{1.2}\right)\left(\frac{4.2}{0.7}\right)\left(\frac{5/1000}{(4)(3)(4)}\right)(4-20)$$

$$= \underline{\underline{11.1\,^\circ C}}$$

(this is high because of our assumptions)

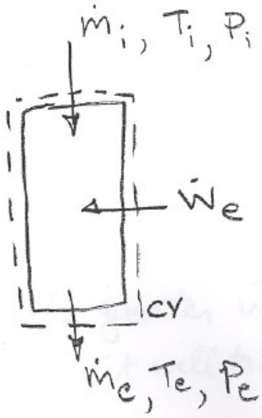
OBLEM 2

A) steady state conditions

CV: the tank boundaries

assumptions:
- $P_e \simeq P_i$ (inlet & outlet pressure equal)
- tank is adiabatic
- negl. ke, pe changes
- water is incompressible

$$\Delta h = \Delta u + \Delta(Pv) \qquad P_e \simeq P_i$$
$$= \Delta u + \underbrace{\cancel{P\Delta v}}_{\text{incomp.}} + \cancel{v\Delta P}$$
$$\Delta h \simeq \Delta u = \underline{c\Delta T}$$



mass: $\dot m_i = \dot m_e = \dot m$

energy: $\underbrace{\cancel{\dot Q}}_{\text{adiabatic}} - \dot W + \dot m_i h_i - \dot m_e h_e = \frac{dU}{dt} = 0$ since steady state

$\dot W = -\dot W_e$ (work out is positive)

$$\Rightarrow \quad \dot W_e = \dot m(h_e - h_i) = \dot m(u_e - u_i + P_i v_i - P_e v_e)$$

but $P_i v_i \simeq P_e v_e$ since $v_i \simeq v_e$, $P_i \simeq P_e$

$$\Rightarrow \quad \dot W_e = \dot m(u_e - u_i) = \dot m c(T_e - T_i) \quad \text{where } T_i = T_c$$

$$\boxed{T_{e,ss} = T_c + \frac{\dot W_e}{\dot m c}}$$

"$T_{e,ss}$" is steady state exit temp.

B) unsteady, zero flow

$T_1 = T_{e,ss}$; $T_2 = T_s$

CV: same as above

assumptions: same as above plus the change in internal energy of the water tank itself is negligible (the mass of the metal $\times$ C_{metal} $\ll$ $M_{water} \times C_{water}$)

mass: $\dot m_i = \dot m_e = 0 \Rightarrow \frac{dm}{dt} = 0$ (mass in the tank is const.)

energy: $\cancel{\dot Q} - \dot W + \cancel{\dot m_i} h_i - \cancel{\dot m_e} h_e = \frac{dU}{dt}$; $\dot W = -\dot W_e$

$$\Rightarrow \dot W_e = \frac{dU}{dt} = m\frac{du}{dt} + u\cancel{\frac{dm}{dt}}^{0} \Rightarrow \dot W_e = m\frac{du}{dt}$$

$$\dot{W}_e = m\frac{du}{dt}$$

$$\dot{w}_e = \rho_w \mathcal{V} c \frac{dT}{dt}$$

$$\frac{\dot{w}_e}{\rho_w \mathcal{V} c}\, dt = dT \Rightarrow \frac{\dot{w}_e}{\rho_w \mathcal{V} c} \Delta t = \Delta T$$

$$\boxed{\Delta t = (T_s - T_{e,ss}) \frac{\rho_w \mathcal{V} c}{\dot{w}_e}}$$

this makes sense: the greater $\dot{w}_e$, the less time it will take

$$\left(= (T_s - T_c)\frac{\rho_w \mathcal{V} c}{\dot{w}_e} - \frac{\rho_w \mathcal{V}}{\dot{m}_{ss}} \right)$$

c) unsteady, with flow

CV: same as above

assumptions: as a) & b) plus T exiting the tank is equal to T within the tank, which is uniform everywhere

$$\cancel{\dot{Q}} - \dot{W} + \dot{m}h_i - \dot{m}h_e = \frac{dU}{dt} = mc\frac{dT}{dt} \; ; \; \dot{W} = -\dot{W}_e$$

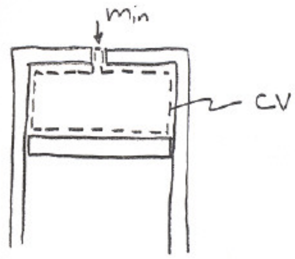
$$\dot{W}_e + \dot{m}cT_i - \dot{m}cT_e = mc\frac{dT}{dt} \; ; \; T_i = T_c \; ; \; T_e = T$$

$$\Rightarrow \; mc\frac{dT}{dt} + \dot{m}cT - \dot{m}cT_e - \dot{W}_e = 0$$

$$\boxed{\rho_w \mathcal{V} c \frac{dT}{dt} + \dot{m}cT - \dot{m}cT_c - \dot{w}_e = 0}$$

1st order linear d.e.

PROBLEM 3



CV: boundaries shown

Assumptions: • quasi-steady

→ small hole allows air to leak in slowly as piston falls. entire system is at the same pressure

• negligible ΔPE, ΔKE of CV

mass: $m_{in} - \cancel{m_{out}} = \Delta m = m_2 - \cancel{m_1}$

$\Rightarrow m_{in} = m_2$

energy: $\cancel{Q} - W + m_{in}h_{in} - \cancel{m_{out}}h_{out} = \Delta E = \Delta U = m_2u_2 - \overset{0}{\cancel{m_1}}u_1$

$$-W + m_{in}h_{in} = m_2u_2 \; ; \; m_{in} = m_2 = m_f$$

but $-W = -\int P\,dV = -P_0(V_f - \cancel{V_1}) = -P_0V_f$

$\Rightarrow -P_0V_f = m_f(u_f - h_{in})$; $m_f = \frac{P_0V_f}{RT_f}$ (ideal gas law)

$-P_0V_f = \frac{P_0V_f}{RT_f}(c_vT_f - c_pT_0)$ since $h_{in} = h_0 = c_pT_0$

$RT_f = c_pT_0 - c_vT_f$

$(R + c_v)T_f = c_pT_0$

but $R + c_v = c_p$ (ideal gas)

$\Rightarrow c_pT_f = c_pT_0 \Rightarrow \boxed{T_f = T_0}$

the enthalpy carried into the cylinder through the hole went into doing boundary work ($\int P_0\,dV$) and increasing to the internal energy of the cylinder from zero to

$m_fu_f = m_{in}u_0 \Rightarrow h_0 = u_0 + P_0v_0$

went into the cylinder — went to boundary work